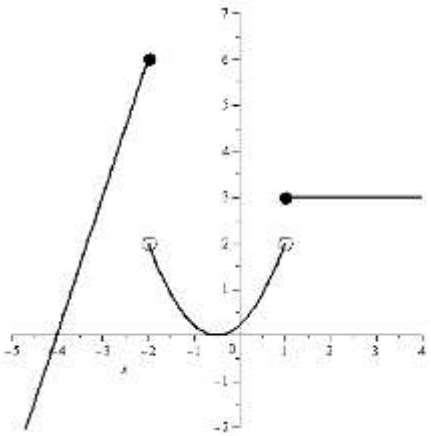
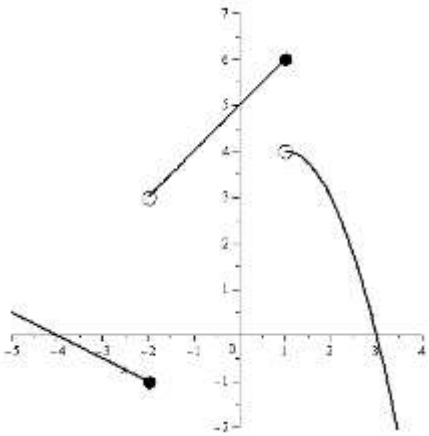
Continuity Homework

1. Consider the functions *f* and *g* shown below. (The function *f* is shown on the left and the function *g* is shown on the right.)





   a. Is the function $h(x) = f(x) + g(x)$ defined at *x* = -2? If so, what is *h* (-2)?

   b. Does $\lim_{x \to -2} \left(f(x) + g(x) \right)$ exist? If so, what is it?

   c. Is the function (*f* +*g*) continuous at *x* = -2? Explain.

   d. Is the function $h(x) = f(x)g(x)$ continuous at x = 1? Explain.

   e. Is the function $h(x) = \dfrac{f(x)}{g(x)}$ continuous at x = -2? Explain.

2. Is it possible to find a constant *a* such that $g(x) = \begin{cases} \dfrac{a}{x} & \text{for } x < 5 \\ -1 & \text{for } x = 5 \\ \dfrac{x}{a} & \text{for } x > 5 \end{cases}$ is continuous at *x* = *5*?

   Explain.

3. Is it possible to find constants *a* and *b* such that $g(x) = \begin{cases} x^3 & \text{for } x < -1 \\ ax + b & \text{for } -1 \le x < 1 \\ x^2 + 2 & \text{for } x \ge 1 \end{cases}$

   is continuous for all *x*? Explain.

4. Let $f(x) = \begin{cases} ax^2 + 3 & \text{for } x < 2 \\ 3x - 5 & \text{for x } \geq 2 \end{cases}$.

a. Find *a* so that f is continuous at x = 2.

b. For that value of *a*, is f differentiable at x = 2? Justify your answer using the definition of the derivative.